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A FEW MORE MECHANICAL-FLIGHT FORMULAS
WITHOUT THE AID OF POLAR DIAGRAMS

By Martin Schrenk

From the 1927 Yearbook of the
"Deutsche Versuchsanstalt für Luftfahrt"

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I. Introduction

In the Seventy-Fourth Report of the D.V.L. ("Deutsche Versuchsanstalt für Luftfahrt"), the writer proposed a method for calculating flight performances without using independent coefficients (c_a , c_w , c_w/c_a , $c_w/c_a^{3/2}$). These expressions will here be amplified and supplemented. The calculation of the climbing and speed performances was carried out on the assumption of constant profile drag, whereby there was assumed a logical separation of the airplane resistances into one part which is connected with the dynamic pressure (head resistance or drag) and into another part which is connected with the reciprocal of the dynamic pressure (induced drag). These calculations will here be continued by the investigation of the relations in flight with the best coefficient of glide (L/D ratio), whereby it is shown that the expressions obtained differ only by numerical factors from the ones calculated for the best climbing conditions. In pursu-

*"Einige weitere flugmechanische Beziehungen ohne Zuhilfenahme der Polare," the Seventy-Ninth Report in the 1927 Yearbook of the "Deutsche Versuchsanstalt für Luftfahrt," pp. 145-151.

This report is supplementary to Report Seventy-Four of the "D.V.L.": Calculation of Airplane Performances without the Aid of the Polar Diagrams, by the same author. (See N.A.C.A. Technical Memorandum No. 456.)

ance of this subject, it is found that the performance characteristics of the airplane (L/D ratio, drag, vertical speed of descent, minimum power required to maintain horizontal flight) follow a law independent of the shape of the airplane, when based on the flight conditions at the best L/D ratio.

The reliability of the assumption of a parabolic shape of the polar curve is investigated and found satisfactory for all practical purposes. It is further shown that the aerodynamically best possible or "ideal" airplane is produced on this assumption.

Lastly, detailed suggestions are given on the possibilities of application of this method of calculation. It especially simplifies the design and evaluation of structural changes and the determination of the limits of technical possibilities.

The present report deals only with the relations of the airfoils. The mutual action of the airfoils, engine and propeller will be treated in a subsequent report.

II. Symbols

1. Airplane without Power Plant

G, full load, kg.

F, wing area, m².

b, span, m.

t, mean chord (F/b), m. On a biplane this is the sum of the mean upper and lower chords divided by the span of the longer wing $\frac{F_o + F_u}{b}$.

- f_w , equivalent flat-plate areas (W/q), m^2 :
 f_{wp} , equivalent flat-plate area of wings, m^2 ;
 f_{wr} , equivalent flat-plate area of non-lifting parts, m^2 ;
 f_{ws} , total equivalent flat-plate area ($f_{wp} + f_{wr}$), m^2 ;
- κ , reduction coefficient (W_D/W_E) for the induced drag of a biplane, as compared with a monoplane of like span (according to Prandtl).*
- c_a , lift coefficient.
- c_w , drag coefficient (c_{wp} , c_{wr} , c_{ws} , c_{wi}).

2. Power Plant

- N , engine power, HP.
- η , propeller efficiency (referred to gliding-flight polar with like c_a).

3. Performances

- v , horizontal speed, m/s.
- w_s , vertical speed of descent, m/s.
- H , altitude of flight, km.
- ρ , air density (γ/g) $kg\ s^2/m^4$.
- q , dynamic pressure ($\rho v^2/2$), kg/m^2 .

The subscripts denote:

- σ , sea level;
- a , critical altitude (up to which the engine power does not diminish);
- g , ceiling;
- c , flight condition of best L/D ratio;

*"Ergebnisse der Aerodynamischen Versuchsanstalt zu Gottingen," Report II, p. 9 ff.

- i, quantities corresponding to the induced drag of equivalent monoplanes (for example, b_i "induced span").

III. Flight at Best L/D Ratio

1. Derivation of Formulas

a) The flight condition of best L/D ratio (glide coefficient) is characterized by the smallest possible total resistance or drag. This condition is utilized in calculating the performance values. It is (total drag = structural drag + induced drag)

$$W = q f_{ws} + \frac{\kappa G^2}{\pi q b^2} \quad (1)$$

or

$$W = \frac{\rho}{2} v^2 f_{ws} + \frac{2}{\pi \rho} \left(\frac{G}{b_i} \right)^2 \frac{1}{v^2} \quad (2)$$

if $b/\sqrt{\kappa} = b_i$ denotes the "induced span" (span of the equivalent monoplane with respect to the induced drag). The minimum value of W is obtained from the condition

$$\frac{dW}{dv} = \rho v f_{ws} - \frac{4}{\pi \rho} \left(\frac{G}{b_i} \right)^2 \frac{1}{v^3} = 0$$

to

$$v_c = \left(\frac{4}{\pi} \right)^{1/4} \rho^{-1/2} \left(\frac{G}{b_i} \right)^{-1/2} f_{ws}^{-1/4} \quad (3)$$

This is the speed along the flight path at the best L/D ratio. Equations (2) and (3) yield the minimum drag.

$$W_{\min} = \left(\frac{4}{\pi} \right)^{1/2} \frac{G}{b_i} f_{ws}^{1/2} \quad (4)$$

The best L/D ratio is therefore

$$\epsilon_{\min} = \frac{W_{\min}}{G} = \left(\frac{4}{\pi}\right)^{1/2} \frac{f_{ws}^{1/2}}{b_i} \quad (5)$$

The corresponding speed of vertical descent is

$$w_{se} = v_{\epsilon} \epsilon_{\min} = \left(\frac{4}{\pi}\right)^{3/4} \rho^{-1/2} \left(\frac{G}{b_i}\right)^{1/2} \frac{f_{ws}^{1/4}}{b_i} \quad (6)$$

Moreover, the energy consumption required to maintain flight at the best L/D ratio is

$$N_{se} = \frac{W_{\min}}{75} v \left(\frac{4/\pi}{75}\right)^{3/4} \rho^{-1/2} \left(\frac{G}{b_i}\right)^{3/2} f_{ws}^{1/4} \quad (7)$$

The air density and the altitude can be easily calculated with the aid of the power required to maintain horizontal flight, if the exponential law, deduced in the previous article, is used for the decrease in engine power with the air density. It was

$$\frac{N_{\epsilon}}{N_a} = \left(\frac{\rho_{\epsilon}}{\rho_a}\right)^{1.4}, \quad (8)$$

whereby the subscript a denotes the condition at the critical altitude of flight; and ϵ , at the altitude attainable with the best L/D ratio. (It must be borne in mind that the engine power is assumed to remain constant from the ground up to the critical altitude.)

The power required to maintain horizontal flight at any altitude equals the propeller performance

$$\left(\frac{4/\pi}{75}\right)^{3/4} \rho_{\epsilon}^{-1/2} \left(\frac{G}{b_i}\right)^{3/2} f_{ws}^{1/4} = N_{\epsilon} \eta_{\epsilon} = N_a \eta_{\epsilon} \left(\frac{\rho_{\epsilon}}{\rho_a}\right)^{1.4}.$$

from which is derived the air-density ratio

$$\frac{\rho_a}{\rho_c} = 8.90 \left(\frac{N_a \eta_c \rho_a^{1/2}}{\left(\frac{G}{b}\right) f_{wsi}^{3/2} 1/4} \right)^{0.53} \quad (9)$$

With the law for the dependence of the altitude on the air density (also previously employed),

$$H = 20.9 \lg \frac{\rho_0}{\rho_H} \quad (10)$$

which represents a close approximation up to 10,000 m (32,800 ft.), there is deduced from equation (9) the formula for the flight altitude attainable with the best L/D ratio:

$$H_c = H_a + 11.0 \lg \frac{62.5 N_a \eta_c \rho_a^{1/2}}{\left(\frac{G}{b_i}\right)^{3/2} f_{ws}^{1/4}} \quad (11a)$$

or

$$H_c = H_a + 19.8 - 11.0 \lg \frac{\left(\frac{G}{b_i}\right)^{3/2} f_{ws}^{1/4}}{N_a \eta_c \rho_a^{1/2}} \quad (11b)$$

b) The most important fundamental result of these calculations lies in the knowledge that all the formulas differ by only a single numerical factor (or a constant quantity) from the formulas previously deduced for the best coefficient of climb, and that therefore the flight speed, climbing speed, L/D ratio, drag, and required power for both flight conditions always stand in a constant ratio, when the profile-drag coefficient remains constant. This condition will be considered again later.

c) All the formulas are repeated in the following table. The last column shows the mutual relations, which can always be expressed as powers of the number 3.

TABLE

		F o r m u l a s
Flight-path speed, m/s	v	$a \rho^{-1/2} \left(\frac{G}{b_i f_{ws}^{1/2}} \right)^{1/2}$
Vertical speed of descent, m/s	w_s	$a \rho^{-1/2} \frac{G^{1/2} f_{ws}^{1/4}}{b_i^{3/2}}$
Total drag, kg	W	$a \frac{G}{b_i} f_{ws}^{1/2}$
L/D ratio	ϵ	$a \frac{f_{ws}^{1/2}}{b_i}$
Power required for horizontal flight, HP.	N_s	$a \rho^{-1/2} \left(\frac{G}{b_i} \right)^{3/2} f_{ws}^{1/4}$
Air density ratio	$\frac{\rho_a}{\rho_H}$	$a \left(\frac{N_a \eta_H \rho_a^{1/2}}{\left(\frac{G}{b_i} \right)^{3/2} f_{ws}^{1/4}} \right)^{0.53}$
Ceiling, km	H	$H_a + a - \lg \frac{\left(\frac{G}{b_i} \right)^{3/2} f_{ws}^{1/4}}{N_a \eta_H \rho_a^{1/2}}$
Lift coefficient	c_a	$\frac{a}{\kappa^{1/2}} \frac{f_{ws}^{1/2}}{t}$
Coefficient of climb	$\frac{c_w}{c_a^{3/2}}$	$a \frac{f_{ws}^{1/4} t^{1/2} \kappa^{3/4}}{b}$

TABLE (Cont.)

		Coefficient a		R a t i o (1):(2)
		(1) At best coefficient of climb	(2) At best L/D ratio	
Flight-path speed, m/s	v	0.81	1.06	$1:3^{1/4}=0.76$
Vertical speed of descent, m/s	w_s	1.05	1.20	$2:3^{3/4}=0.88$
Total drag, kg	W	1.31	1.13	$2:3^{1/2}=1.16$
L/D ratio	ϵ	1.31	1.13	$2:3^{1/2}=1.16$
Power required for horizontal flight, HP.	N_s	$\frac{1}{71.3}$	$\frac{1}{62.5}$	$2:3^{3/4}=0.88$
Air density ratio	$\frac{\rho_a}{\rho_H}$	9.55	8.90	$(2:3^{3/4})^{0.53} = 0.935$
Ceiling, km	H	20.4	19.8	$(1)-(2)=0.6 \text{ km}$
Lift coefficient	c_a	3.07	1.77	$3^{1/2}:1 = 1.73$
Coefficient of climb	$\frac{c_w}{c_a^{3/2}}$	0.75	0.85	$2:3^{3/4} = 0.88$

2. Validity Limits of the Formulas

The derivation of the formulas proceeds from the assumption that the total equivalent flat-plate areas (profile drag + structural drag) remain constant throughout the whole range of the angles of attack on normal flight (Fig. 1). This assumption must be verified and the effect of the deviations estimated.

a) Among the nonlifting parts of the airplane the fuselage plays the most important role. All other parts are either so small or so shaped (e.g., the landing wheels) that moderate changes in the angle of attack do not affect the drag values.

Considered by themselves, ordinary fuselages show a moderate drag increase for a large deviation of the angle of attack from the line of symmetry. In the presence of the wings, however, the mutual effect is usually so great that correct results can be obtained only by simultaneous tests of the fuselage and wings. Such tests lead to the conclusion that, for favorably shaped fuselages, the mean increase in the equivalent flat-plate area due to the fuselage is independent of the angle of attack, excepting for very unfavorable arrangements (e.g., wings located slightly below the fuselage) and perhaps very large fuselages relative to the size of the wings.* In certain positions (high-

*"Ergebnisse der Aerodynamischen Versuchsanstalt Göttingen," Report I, Chapter IV, Section 7. Unpublished experiments performed at Göttingen in 1934, according to the instructions of the writer, on two models of the Daimler L 20 (both high-wing and low-wing) lead to the same conclusion.

wing monoplane) the apparent equivalent flat-plate area of the fuselage can even grow smaller with increasing angle of attack.

b) The profile drag of the wing itself is largely dependent on Reynolds Number and on the roughness of the surface. Figure 2 shows the profile-drag coefficients of a thick Junkers wing ($d : t = 0.18$) for various degrees of doping and polishing of the covering fabric.* It is seen that, with the smoothest surface, the drag coefficient of this profile is actually constant over a wide c_a range.

c) Nevertheless there remains, for all the profiles, a great increase in the drag coefficient in the vicinity of the maximum lift, produced by the gradual separation of the boundary layer. Hence, in the previous article, an auxiliary method was given for the case when the best c_a in climbing under the assumption of parabolic polars is considerably greater than 1, i.e., when it lies in the domain of great profile-drag increments. This case occurs with most airplanes.

On the other hand, the lift coefficient in flight at the best L/D ratio is almost always less than 1 (Fig. 3). Here the formulas apply very accurately. This flight condition therefore offers a sure basis for judging flight, even with the best coefficient of climb. Since both flight conditions with parabolic polars stand in certain fixed numerical relations to one another,

*According to experiments by the writer soon to be published.

incontestable limits can be given for the individual power values based on the values at the best L/D ratio. Hence, for ordinary wing sections with the best coefficient of climb, the vertical speed of descent is 88% in the most favorable case, the power required to maintain horizontal flight is the same, and the maximum attainable air density is 93.5% of the corresponding values at the best L/D ratio.

The ceiling is not over 0.6 km (1968 ft.), or in the stratosphere 0.45 km (1476 ft.), higher than at the best L/D ratio.

The best values of these performances lie in fact between the given limits. The corresponding lift coefficients are therefore lower, or the speeds higher, than the values given in table.

When it is considered that (with a suitable propeller) the propeller efficiency still increases somewhat with increasing speed, it is obvious that the best value of the mutual action of the airfoil system and power plant more closely approximates the speed at the best L/D ratio, which therefore grows continually more important.

d) Model experiments on wing sections do not show the constancy of the profile drag so pronounced as the abovementioned tests.*

So long as we depend principally on the latter (in spite of what is said in paragraph c), the need will be felt, under

*Oskar Schrenk, "Systematische Untersuchungen an Joukowski-Profilen," "Zeitschrift für Flugtechnik und Motorluftschiffahrt," May 28, 1927, pp. 225-230, (N.A.C.A. Technical Memorandum No. 442).

some circumstances, of a more accurate comprehension of the relations.

In most instances the profile-drag polars can be replaced in the technically important aviation domain by a parabola symmetrical to the c_w axis (Fig. 3). Since, in this case, the profile drag follows the same law as the induced drag, its variability can be expressed by the introduction of an apparent aspect ratio λ'_i or an apparent induced span b'_i . If c_{wpo} is the constant share for $c_a = 0$ and if $c_{wp1} - c_{wpo}$ is the increase in the profile drag for $c_a = 1$, we then have, after a short deduction process,

$$\lambda'_i = \frac{F}{b_i^2} + \pi (c_{wp1} - c_{wpo}) \quad (12)$$

$$b'^2_i = \frac{F}{\frac{F}{b_i^2} + \pi (c_{wp1} - c_{wpo})} (< b_i^2) \quad (13)$$

Use can be made of the assumptions, wherever it is desired to estimate the limits of the technically possible on the basis of model experiments. We must then lay an enveloping curve on a group of polars of good profiles of varying camber and thickness. If the very thin wing sections are discarded, we then obtain, up to $c_a = 1.2$, a good approximation to the parabola

$$100 c_{wp} = 0.9 + 0.6 c_a^2 \quad (14)$$

The aspect ratio of such a wing increases therefore by the

amount $0.006 \pi = 0.019$ and the constant component of the profile drag is $c_{wpo} = 0.009$.

e) It must be borne in mind that the induced drag itself is not accurately known. In general, we know only its best value, which appears in elliptical lift distribution. The deviations therefrom in actual practice are generally not very great,* though they are appreciable in comparison with the refinements considered in paragraph d.

Since, however, the best value of the induced drag can be obtained by a suitable contour or wing warping, at least for a predetermined flight condition, and the ideal condition is therefore practically attainable,** these relations will not be further considered here. Only as a starting point, Figure 4 gives a comparison of the theoretical values of κ and the values obtained in Göttingen for symmetrical biplanes with rectangular unwarped wings of equal span.

f) If we take gliding-flight polars, measured in flight, as the basis for designs, we can easily replace these by parabolas in almost all cases. We accordingly obtain (corresponding to the process in paragraph d) an "apparent aspect ratio"

$$\lambda'_i = \pi (c_{w1} - c_{w0}) \quad (12a)$$

* For monoplanes, Betz, "Tragflügeltheorie," "Beiheft (supplement) 2 of the "Z.F.M."; for biplanes, "Messungen an Doppeldeckern," "Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen," Report II.

** With the possible exception of the influence of the fuselage in biplanes, whereon data are yet lacking.

and an "apparent induced span"

$$b_i'^2 = \frac{F}{\pi (c_{wi} - c_{wo})} \quad (13a)$$

c_{ws} is then the distance of the vertex of the parabola from its origin.

In this way the results of a flight test can be expressed in the form of span and total equivalent flat-plate area, an approximation which is not quite true to reality, but which may be of practical advantage.

IV. Introduction of the "Best Gliding Speed"

1. Derivation

If the speed required to maintain the flight condition of the best L/D ratio is termed the "best gliding speed" v_c , we then have, according to formula (3)

$$v_c = \frac{1.06}{\rho^{1/2}} \left(\frac{G}{b_i f_{ws}^{1/2}} \right)^{1/2}$$

a value, which can be immediately calculated for any air density, provided the values of G , b_i , and f_{ws} are known. If this speed v_c is introduced into the general formula (2) for the total drag, we obtain, after a few transformations,

$$W = 0.565 \frac{G}{b_i} f_{ws}^{1/2} \left(\frac{v}{v_c} \right)^2 + \left(\frac{v_c}{v} \right)^2 \quad (14)$$

While formula (2) gives only a general relation between

speed along the flight path and air resistance, formula (14), as an independent variable, contains the ratio of the momentary to the best horizontal speed. Since the part before the brackets is just half the minimum drag, we have

$$W = \frac{W_{\min}}{2} \left(\frac{v}{v_{\epsilon}} \right)^2 + \left(\frac{v_{\epsilon}}{v} \right)^2 \quad (15)$$

In a simple manner we further obtain

$$\epsilon = \frac{\epsilon_{\min}}{2} \left(\frac{v}{v_{\epsilon}} \right)^2 + \left(\frac{v_{\epsilon}}{v} \right)^2 \quad (16)$$

$$W_S = \frac{W_{S\epsilon}}{2} \left(\frac{v}{v_{\epsilon}} \right)^3 + \frac{v_{\epsilon}}{v} \quad (17)$$

$$N_S = \frac{N_{S\epsilon}}{2} \left(\frac{v}{v_{\epsilon}} \right)^3 + \frac{v_{\epsilon}}{v} \quad (18)$$

2. Confirmation

It can be easily seen that this surprisingly simple relation between the performance values and the ratio of any speed along the flight path to the best horizontal speed follows from the assumption of a parabolic polar. Proceeding from this polar, we obtain

$$\frac{c_w}{c_{w\epsilon}} = \frac{c_{ws} + k c_a^2}{c_{ws} + k c_{a\epsilon}^2},$$

in which $k = \frac{\kappa}{\pi} \frac{F}{b^2}$. In consideration of the circumstance that, at the best L/D ratio, the shares of the head resistance and induced drag are equal* ($c_{ws} = k c_{a\epsilon}^2$), we obtain

*Due to a simple geometric relation for the parabola.

$$\frac{c_w}{c_{w\epsilon}} = \frac{1}{2} \frac{c_a}{c_{a\epsilon}} \left(\frac{c_a}{c_{a\epsilon}} + \frac{c_{a\epsilon}}{c_a} \right)$$

and lastly

$$\frac{c_w}{c_a} = \frac{1}{2} \frac{c_{w\epsilon}}{c_{a\epsilon}} \left(\frac{c_a}{c_{a\epsilon}} + \frac{c_{a\epsilon}}{c_a} \right) \quad (16a)$$

the latter being identical with formula (16).

Considered by itself, any other lift coefficient or speed connected with the polar by a geometric relation could be chosen as a reference quantity, but the condition of best L/D ratio yields especially simple relations because of the equality of the two drag components. Moreover, it is technically the most important, because it furnishes the basis for the evaluation of the aerodynamic fineness of the airplane.

3. Application

The relations represented by formulas (15)-(18) are plotted in Figure 5. It is, to a certain extent, a "standardized" representation of the "ideal aircraft" for the L/D ratio, drag, vertical speed of descent and required power, based on the values at the best L/D ratio.

These normal curves of the performance values of airplanes represent a great simplification for the plotting of the performance diagrams. Above all, it is possible in this way to obtain a quick survey of the relations over the whole flight range, independently of the airplane type. Only the relations at the best gliding speed are affected by the values of G , b_1 , and f_{ws} .

More may be said later on the other possible applications of such a "standardized" performance diagram.

4. Determination of the Values v_{ϵ} and ϵ_{\min} Dependent on the Form of the Airplane

It is seen that the calculation of the flight performances is divided into two parts:

a) Calculation of the flight performances w_{ϵ} , ϵ_{\min} (W_{ϵ} , $w_{s\epsilon}$) and $N_{s\epsilon}$ at the best L/D ratio.

b) Use of the "normal" diagram (i.e., the one based on the condition at the best L/D ratio), which can be drawn once for all for the given airplane. The problem b has already been considered in Section III.

We must proceed from v_{ϵ} and ϵ in the determination of the performance values. Formulas (3) and (5) for these values enable the construction of a very simple nomogram (Fig. 6). The further values are then

$$W_{\epsilon} = G \epsilon_{\min} \quad (19)$$

$$w_{s\epsilon} = v_{\epsilon} \epsilon_{\min} \quad (20)$$

$$N_{s\epsilon} = \frac{W_{\epsilon} v_{\epsilon}}{75} = \frac{G v_{\epsilon} \epsilon_{\min}}{75} = \frac{G w_{s\epsilon}}{75} \quad (21)$$

and can be quickly calculated with a slide rule.

5. Graphic Explanation

The "best gliding speed" according to formula (3) and the corresponding "best L/D ratio" (formula (5)) will be considered in greater detail. v_c is determined by the expression

$$\frac{G}{b_i f_{ws}^{1/2}} .$$

This is a wing loading. The area supporting the full load G is obtained from the induced span and the side of the square equivalent to the sum of all the equivalent flat-plate areas, including profile drag (Fig. 7). The greater this area, the smaller the speed of the best L/d ratio.* Hence it is called "speed area." It is equally important with the wing loading in ordinary calculations. The aspect ratio of this "speed area" yields, however, when multiplied by 1.13, the best L/D ratio.

Through these two quantities ϵ and v_c we obtain a new principle of classification, which renders it possible to classify airplanes according to their specific speed (as viewed from the standpoint of their airfoil systems) and according to their economy.

Figure 8 represents several airplanes of very different types, as classified from this viewpoint.** The diagram shows,

*If v_c is compared with the final velocity of free fall, which would be attained by such a speed area loaded with G with $c_w = 1$, this area is found to be $\pi^{1/4} v_c$, i.e., v_c is 3/4 of this falling velocity.

**Largely on the basis of accurate flight measurements. The induced span was taken for all airplanes according to the theoretical optimum. The values serve only as examples.

beside the lines of like vertical speed of descent ($v_c \epsilon_{\min}$) several hyperbolas of like ratio v_c/ϵ_{\min} of both reference sizes.*

It is obvious that four of the otherwise very different airplanes are approximately equivalent with respect to this ratio, while the fifth is considerably higher. The latter airplane was in fact constructed with exceptional care with respect to head resistance and its performances represent considerable technical progress.

V. Conclusions

1. On the Limits of Similarity Considerations

a) Similarity or model laws find many applications in the mechanics of flight. Newton's general similarity law furnishes the basis for the quadratic law of dynamic air forces. Reynolds' similarity law includes the viscosity of the fluid, in addition to the inertia, and enables a statement thereon when the flow picture is similar. Hence the application of Newton's model law is strictly accurate. Here the geometric similarity of the compared objects is always assumed. The nondimensional coefficients for the air forces are then approximately independent of the size of the model and the polar remains constant.

b) Similarity considerations in the design and enlargement

* v_c/ϵ_{\min} indicates the speed of an airplane with $\epsilon_{\min} = 1$, which would be equivalent with respect to the ratio v_c/ϵ_{\min} , hence a sort of "economical speed of comparison."

of airplanes often proceed from the geometric similarity of the airplane. They have often been successfully applied (Lanchester, Rohrbach) and can be of great service. There is danger, however, (as demonstrated by the prevalence of certain erroneous views)* that more or less conscious use of similarity considerations is made in cases where there is no geometric similarity.

In fact, airplanes of different sizes and uses differ greatly from one another in their geometric relations. The size of the non-lifting parts is determined chiefly by the total weight (full load), including the fuselage, landing gear and floating gear, and their economical disposal is determined by the arrangement and size of the wings. The size of the wings, on the contrary, is determined chiefly by the landing speed (wing loading), structural strength, maneuverability, and stowing (span). Attention is called, e.g., to the contrast between a single-seat pursuit airplane, which has relatively small wings, and a large commercial airplane, which carries engines and "useful load" in its wings.

c) If geometric similarity can be assumed, the wing and power loading, as well as the power per unit of wing area, give a good idea of the flight performances to be expected. If such is not the case, however, the aerodynamic coefficients (L/D ratio, coefficient of climb and coefficient of drag in horizontal flight)

*For example, that a great wing loading is unavoidable for attaining high speed, or that great reserve power is possible only with small power loading (both of which views have been controverted by good light airplanes).

must then also be taken into consideration. The relations are thus complicated, however. The coefficients depend on the type of airplane. In particular, the effect of altering the size and shape of the wings can not be disregarded. Altering the chord produces quite a different effect from altering the span. If it is not desired to calculate every example (which is not usually done, due to the time required), the only resource is rough estimation.

The present method avoids this obstacle. It is free from coefficients whose use depends on the assumption of geometric similarity (L/D ratio, coefficient of climb and especially independent coefficients). It is strictly limited to the quantities G , b_i , and f_{ws} , as the origin for all computations.*

d) Thus we obtain a series of special model laws, which are strictly valid for the assumption of parabolic polars.

1. The best L/D ratio of an airplane is constant, when the ratio of the square root of the total equivalent flat-plate area to the induced span is constant.

2. The best horizontal speed does not change, so long as the "speed wing loading" remains constant.

3. The lift coefficient at the best L/D ratio does not change, if the ratio of the square root of the head-resistance

*The ratio G/b_i increases, with geometrically similar enlargement and constant wing loading, linearly with the dimensions and cannot therefore be utilized in comparisons, but G/b_i^2 might be used.

area to the chord (on multiplanes "induced chord" $\kappa^{1/2} t$) remains constant.

4. When based on the condition of best gliding speed, the ratio between the speed and performance values of the airplane without power plant are independent of the shape of the airplane.

The above laws express in words, what the formulas in Sections III and IV express in figures.

2. A Few Possibilities for Applying this Method

This method seems to be especially applicable to the following purposes:

a) Complete details have already been given on the simplification of the design calculations and the facilitation of the apprehension of the structural possibilities.

b) The facilitation of the estimation of the effects of structural alterations of a finished airplane is connected with the above.

c) The judging of the aerodynamic effects of unusual structural details (slotted wings, removal of boundary layer by suction, etc.) is considerably simplified.

d) The determination of the limits of the technical possibilities. For the maximum performances in endurance flight, speed and "short-range flight," such widely differing structural

forms are necessary, when the assumption of geometric similarity no longer suffices. In the use of structural characteristics (weight, span, and total equivalent flat-plate area), one is entirely freed from this assumption and can substitute the maximum or most favorable value for each of these quantities in the corresponding case.

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Complete Legends

Fig. 1. Parabolic and actual polar curves.

The best coefficient of climb lies at the point of contact of the curve $c_w/c_a^{3/2} = \text{constant}$ with the polar. The two coefficient-of-climb curves for parabolic or actual polars differ but little. The best L/D ratios for the actual profile and the parabolic substitute generally coincide.

Fig. 2. Profile-drag polars.

The polars are the result of flight tests with a thick Junkers wing section by the impulse method. They represent the effect of the varying surface treatment of airplane linen on the profile drag.

Fig. 3. Replacement of polar of Göttingen profile 387 by a parabola.

The parabola coincides for a long distance with the polar of this good medium-thick profile. The best L/D ratios at $c_{wr} = 0.03$ (mean ratio) still coincide for the parabola and the actual polar and differ but very little at $c_{wr} = 0.06$ (a very unfavorable value).

Fig. 4. Theoretical and experimental reduction factors κ for symmetrical biplanes.

The values are taken from the Göttingen "Ergebnisse," Report II. They show that, for a symmetrical biplane with rectangular wings, the practical values of κ fall 4-9% short of the theoretically best values.

Fig. 5. "Standardized" performances of an "ideal airplane" (without power plant).

The L/D ratio, total drag, vertical speed of descent and power required to maintain horizontal flight are plotted against the flight speed in relation to the best gliding speed. The curves are strictly accurate, when the profile drag remains constant. The deviations due to an increase in the profile drag at great lift are not very great, however. The deviations of ϵ and W are introduced for the usual relations. The figure shows moreover, the components of the induced drag or induced power required to maintain horizontal flight, which decrease very abruptly with increasing flight speed.

Fig. 6. Nomogram for the best gliding speed v_c and the best L/D ratio ϵ_{min} .

The nomogram, which was made out on the plan of logarithmic rectangular nomograms shows, on the left, the "reduced" full load $G \rho_0/\rho$; on the right, the best gliding speed v_c , above and below the induced span b_i or the total equivalent flat-plate area f_{ws} and, lastly, in the middle, the best L/D ratio ϵ_{min} . The reading is obtained by applying a rectangular cross, as illustrated by an example.

Fig. 7. The "speed area."

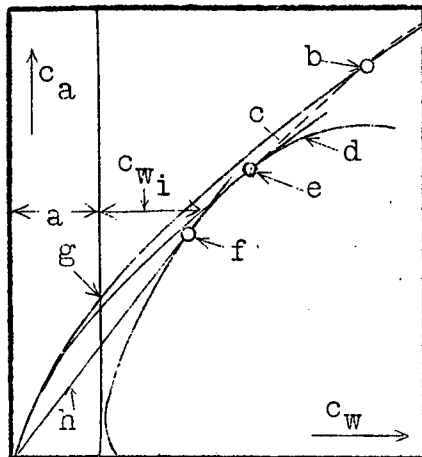
This is a rectangle with the sides b_i and $f_{ws}^{1/2}$. If this imaginary area is charged with the full load G , the load per m^2 furnishes a criterion for the "best gliding speed" v_c . The aspect ratio furnishes a criterion for the best L/D ratio.

Fig. 8. Inverse values of the L/D ratios of various airplanes plotted against their best gliding speeds.

The lines leading to the zero point give the vertical speed of descent at the best L/D ratio. The hyperbolic curves give the "actual relative speed."

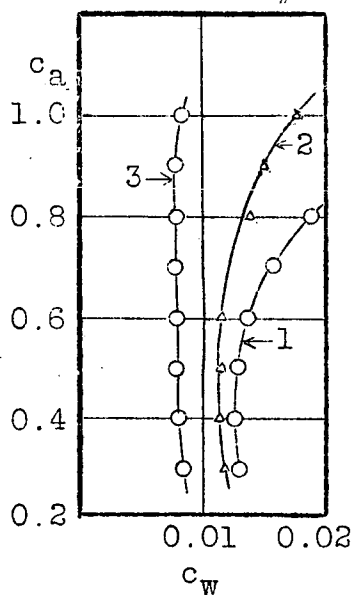
1. Two-seat biplane with high-powered engine.
2. High-wing commercial monoplane with same engine.
3. Two-seat low-wing sport monoplane of usual type.
4. Two-seat low-wing sport monoplane of very fine build.
5. Two-seat low-wing light monoplane.

The dashed arrows indicate a desirable direction of development. The introduced airplanes are only examples, which make no claim to universal validity.



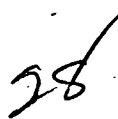
- $a = c_{ws} (c_{wr} + c_{wp})$
 $b =$ Best coefficient of climb. (parabola)
 $c =$ Replacement parabola, (substitute).
 $d =$ Polar.
 $e =$ Best coefficient of climb (actual profile).
 $f =$ Best coefficient of glide, $\frac{L}{D}$
 $g = \frac{c_w}{c_a^{3/2}} = \text{constant.}$
 $h = \frac{c_w}{c_a} = \text{constant.}$

Fig. 1. Parabolic and actual polar curves.

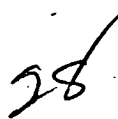


1. Undoped linen.
2. Linen twice doped and lightly polished.
3. Linen doped six times and polished.

Fig. 2. Profile - drag polars.

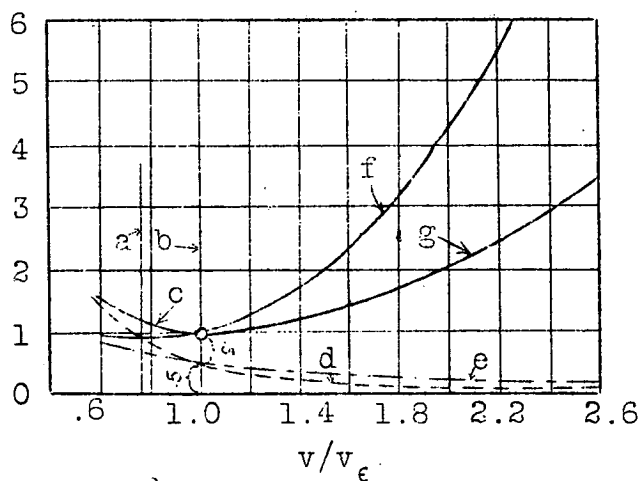


28



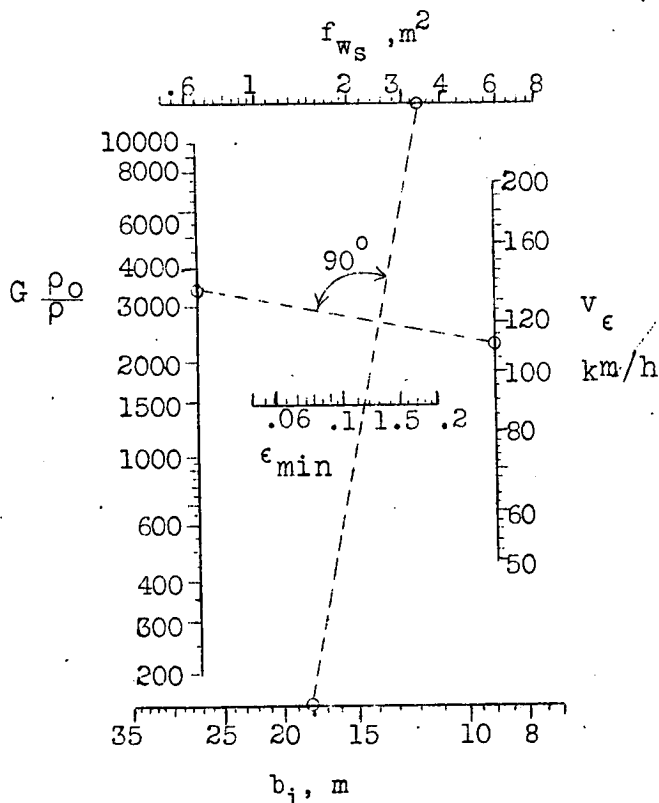
28

$$f = \frac{w_s}{w_{s\epsilon}}, \frac{N_s}{N_{s\epsilon}} \quad g = \frac{\epsilon}{\epsilon_{\min}}, \frac{w}{w_{\min}}$$



- a = best coefficient of climb when $c_{w_0} = \text{const.}$
- b = best coefficient of glide when $c_w = \text{const.}$
- c = for ordinary conditions,
- d = induced drag. (of ϵw).
- e = induced power required to maintain horizontal flight.

Fig.5 "Standardized" performances for the "ideal airplane" (without power plant).



$$\epsilon_{\min} = 1.13 \frac{f_w^{1/2}}{b_i}$$

$$v_\epsilon = \frac{3.82}{\rho^{1/2}} \left(\frac{G}{b_i f_w^{1/2}} \right)^{1/2}$$

Fig.6 Nomogram for the best gliding speed v_ϵ and the best L/D ratio ϵ_{\min} .

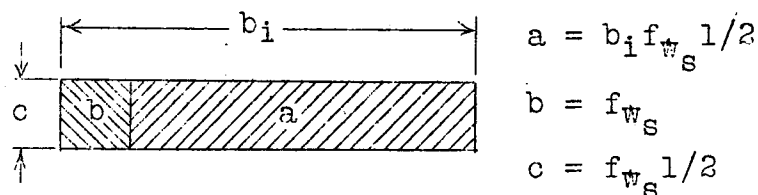


Fig.7 The "speed area".

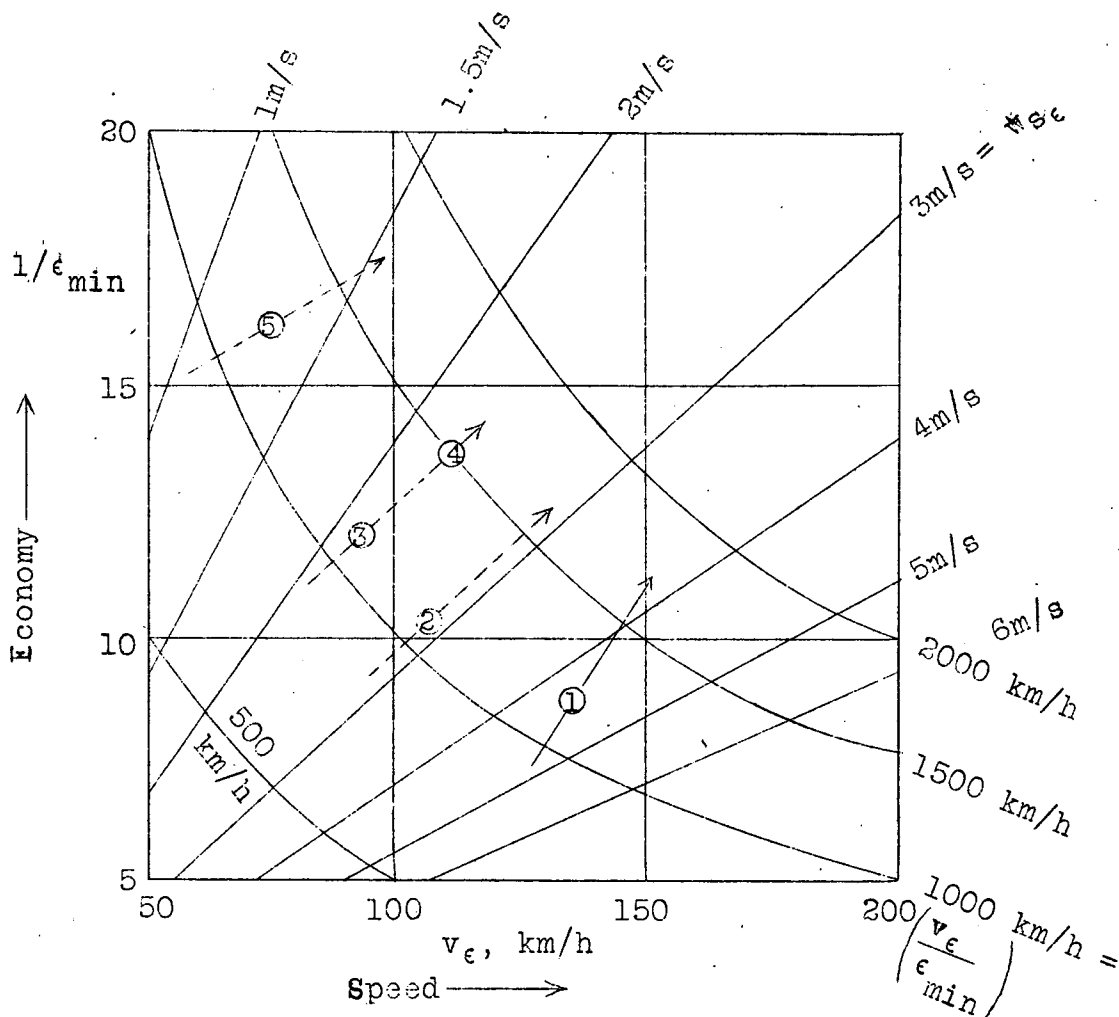


Fig.8 Inverse values of the L/D ratios of various air-planes plotted against their best gliding speeds.